Section 5 / B

Tanta University
Faculty of Engineering
Computers and Control Department

Fourth Year Students
First Term
2012 / 2013

NEURAL NETWORKS

Hyperbolic tangent functions as neuronal signals

Solve the following five problems.

Problem 1

Consider a neuronal signal of the form of a hyperbolic tangent function $h(\alpha x) = \tanh(\alpha x)$, where α is a positive parameter. Verify that

(a) $h(\alpha x) = \dfrac{2}{1+e^{-2\alpha x}} - 1$

(b) $\dfrac{d}{dx}[h(\alpha x)] = \alpha [1 - h^2(\alpha x)]$

(c) $\dfrac{d}{dx}[h(\alpha x)]\Big|_{max} = \alpha$, occurring at $x = 0$

(d) $\dfrac{d^2}{dx^2}[h(\alpha x)] = -2\alpha^2 h(\alpha x)[1 - h^2(\alpha x)]$

Problem 2

A neuron receives two inputs $x_1 = 1.3$ and $x_2 = 1.5$ with inputs $w_1 = 1$ and $w_2 = 2$, respectively. The bias weight w_0 is a design parameter. The neuron employs a hyperbolic tangent function of the form $s = \tanh(0.8y)$, where s is the output signal and y is the activation.

(a) For $w_0 = -2.5$, find s.
(b) For $s = 0.71$, find w_0.

Problem 3

A single neuron receives two inputs $x_1 = 1.5$ and

$x_2 = 2$ with weights $w_1 = 2$ and $w_2 = -1.1$, respectively. The bias weight is $w_0 = 0.4$. The neuron employs a hyperbolic tangent function of the form $s = \tanh(0.75y)$, where s is the output signal and y is the activation.

(a) Find the value of s.

(b) Find the value of the derivative $\frac{ds}{dy}$.

(c) Plot s as a function of y and locate the operating point (y, s).

(d) Plot $\frac{ds}{dy}$ as a function of y and locate the operating point $(y, \frac{ds}{dy})$.

(e) What is the maximum value of $\frac{ds}{dy}$? Where does it occur?

Problem 4

A single neuron receives three inputs $x_1 = 2$, $x_2 = 3$, and $x_3 = 2.5$ with weights $w_1 = 1.2$, $w_2 = -1.1$, and $w_3 = -1$, respectively. The bias weight w_0 is a design parameter. The neuron employs a hyperbolic tangent function of the form $s = \tanh(0.5y)$, where s is the output signal and y is the activation. Find the value of w_0 such that $\frac{ds}{dy} = 0.226$. What are the corresponding values of y and s?

Problem 5

A single neuron receives two inputs $x_1 = 0.8$ and $x_2 = 1.2$ with weights $w_1 = 1.6$ and $w_2 = 0.6$, respectively. The bias weight is $w_0 = -1.4$. The neuron employs a hyperbolic tangent function of the form $s = \tanh(\alpha y)$, where s is the output signal, y is the activation, and α is a positive parameter. The derivative of s with respect to y is found to be 0.311. Calculate the values of α, y, and s.

Prof. Dr. Mahmoud M. Fahmy

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(d) $\dfrac{d^2}{dx^2}[h(\alpha x)] = -2\alpha^2 h(\alpha x)[1-h^2(\alpha x)]$

Solution

(a) $L.H.S. = \tanh(\alpha x) = \dfrac{\sinh(\alpha x)}{\cosh(\alpha x)} = \dfrac{(e^{\alpha x} - e^{-\alpha x})/2}{(e^{\alpha x} + e^{-\alpha x})/2}$

$= \dfrac{1-e^{-2\alpha x}}{1+e^{-2\alpha x}} = \dfrac{2-1-e^{-2\alpha x}}{1+e^{-2\alpha x}} = \dfrac{2}{1+e^{-2\alpha x}} - 1$

$= R.H.S.$

(b) $\dfrac{d}{dx}[\tanh(\alpha x)] = \alpha\,\mathrm{sech}^2(\alpha x) = \alpha[1-\tanh^2(\alpha x)]$
$= \alpha[1-h^2(\alpha x)]$

(c) To obtain the maximum value of the first derivative $\frac{d}{dx}[h(\alpha x)]$, we find an expression for the second derivative $\frac{d^2}{dx^2}[h(\alpha x)]$ and equate it to zero.

$$\frac{d^2}{dx^2}[\tanh(\alpha x)] = \frac{d}{dx}[\alpha\,\mathrm{sech}^2(\alpha x)]$$

$$= -2\alpha^2\,\mathrm{sech}^2(\alpha x)\tanh(\alpha x)$$
$$= 0$$

Since α and $\mathrm{sech}(\alpha x)$ are greater than zero, we

should have

$$\tanh(\alpha x) = 0$$

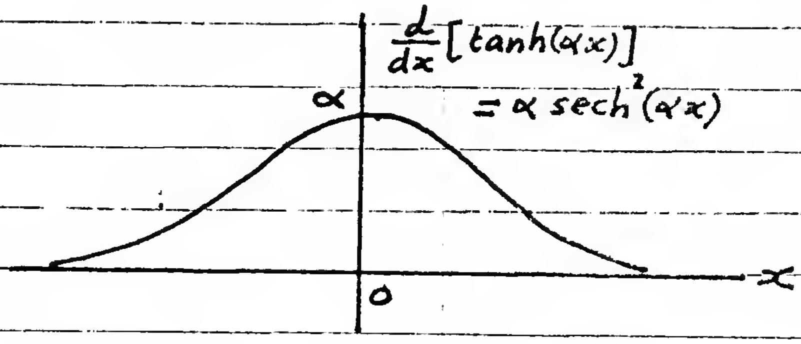
or

$$x = 0$$

Therefore,

$$\frac{d}{dx}\left[\tanh(\alpha x)\right]\bigg|_{max} = \alpha\left[1 - \tanh^2(\alpha x)\right]\bigg|_{x=0}$$

$$= \alpha$$



(d) From the solution of part (c),

$$\frac{d^2}{dx^2}[\tanh(\alpha x)] = -2\alpha^2\, \mathrm{sech}^2(\alpha x) \tanh(\alpha x)$$
$$= -2\alpha^2 \tanh(\alpha x)\left[1 - \tanh^2(\alpha x)\right]$$
$$= -2\alpha^2 h(\alpha x)\left[1 - h^2(\alpha x)\right]$$

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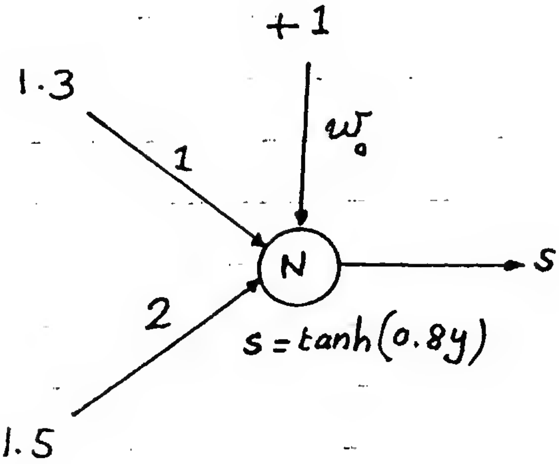

<u>Solution</u>

Activation,

$$y = (1.3)(1) + (1.5)(2) + w_0$$
$$= 4.3 + w_0$$

(a) For $w_0 = -2.5$,

$$y = 4.3 - 2.5 = 1.8$$
$$s = \tanh(0.8 \times 1.8)$$
$$= \underline{0.894}$$

(b) For $s = 0.71$,

$$\tanh(0.8y) = 0.71$$
$$0.8y = \tanh^{-1}(0.71) = 0.887$$
$$y = 1.109$$
$$4.3 + w_0 = 1.109$$
$$w_0 = \underline{-3.191}$$

single neuron receives two inputs $x_1 = 1.5$ and $x_2 = 2$ with weights $w_1 = 2$ and $w_2 = -1.1$, respectively. The bias weight is $w_0 = 0.4$. The neuron employs a hyperbolic tangent function of the form $s = \tanh(0.75y)$, where s is the output signal and y is the activation.

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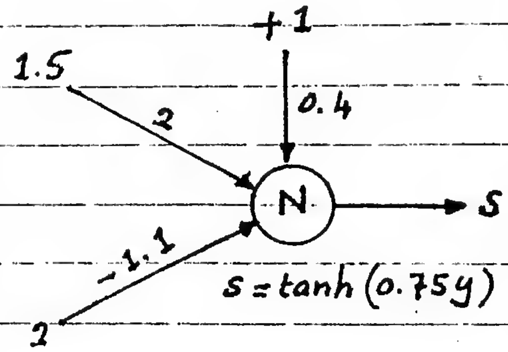


Solution

Activation,

$y = (1.5)(2) + (2)(-1.1) + 0.4$

$= 1.2$

Output signal,

$s = \tanh(0.75 \times 1.2) = \underline{0.716}$

Derivative,

$\frac{ds}{dy} = 0.75 \operatorname{sech}^2(0.75y) = 0.75[1 - \tanh^2(0.75y)]$

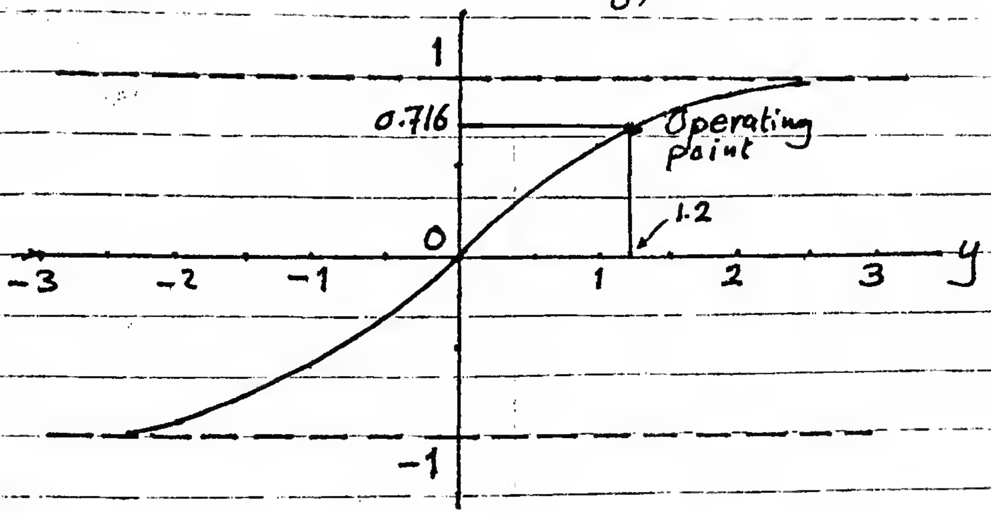
$= 0.75(1 - s^2)$

$= \underline{0.366}$

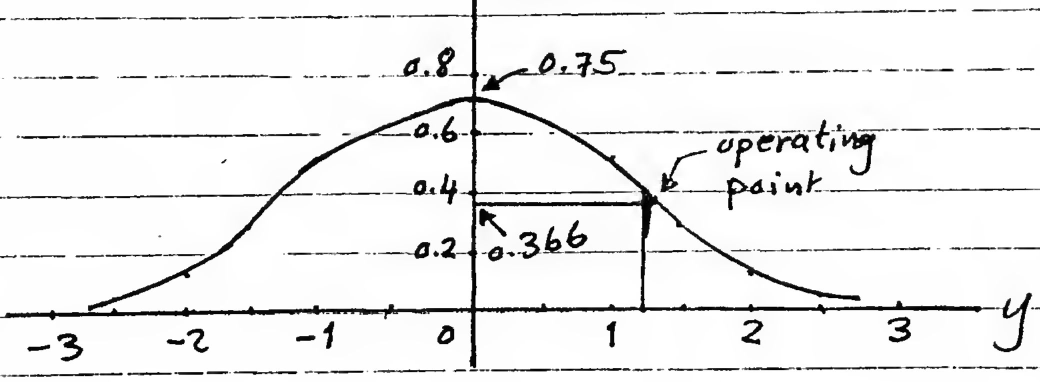
y	-2.5	-2	-1.5	-1	-0.5	0
$s = \tanh(0.75y)$	-0.954	-0.905	-0.809	-0.635	-0.358	0
$\frac{ds}{dy} = 0.75(1 - s^2)$	0.067	0.136	0.259	0.448	0.654	0.75

0.5	1	1.5	2	2.5
0.358	0.635	0.809	0.905	0.954
0.654	0.448	0.259	0.136	0.067

$s = \tanh(0.75y)$

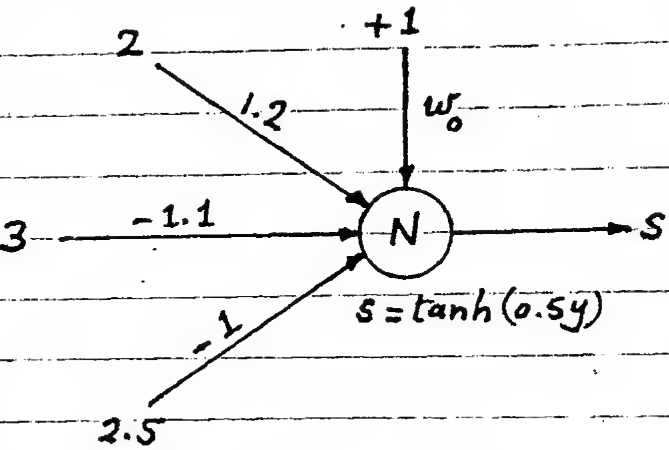


$$\frac{ds}{dy} = 0.75\left[1 - \tanh^2(0.75y)\right]$$
$$= 0.75\,(1 - s^2)$$



Maximum value of $\frac{dy}{ds}$ = $\underline{0.75}$, and occurs at $y = \underline{0}$.

A single neuron receives three inputs $x_1 = 2$, $x_2 = 3$, and $x_3 = 2.5$ with weights $w_1 = 1.2$, $w_2 = -1.1$, and $w_3 = -1$, respectively. The bias weight w_0 is a design parameter. The neuron employs a hyperbolic tangent function of the form $s = \tanh(0.5y)$, where s is the output signal and y is the activation. Find the value of w_0 such that $\frac{ds}{dy} = 0.226$. What are the corresponding values of y and s?



Solution

Activation,

$$y = (2)(1.2) + (3)(-1.1) + (2.5)(-1) + w_0$$
$$= -3.4 + w_0$$

Derivative,

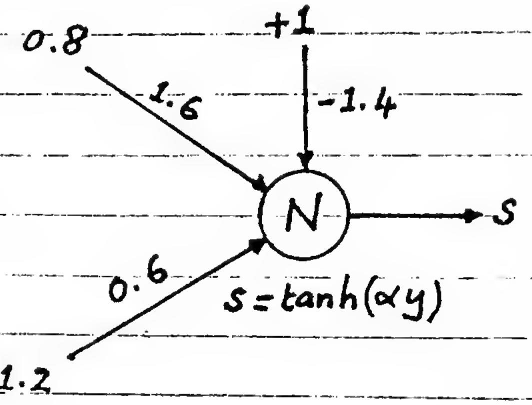
$$\frac{ds}{dy} = 0.5 \operatorname{sech}^2(0.5y)$$
$$= 0.5\left[1 - \tanh^2(0.5y)\right]$$
$$= 0.226$$

$$\tanh^2(0.5y) = 0.548$$
$$\tanh(0.5y) = \sqrt{0.548} = \pm 0.74$$
$$0.5y = \pm 0.95$$
$$y = \pm 1.9$$

For $y = 1.9$, $w_0 = 1.9 + 3.4 = 5.3$, $s = \tanh(0.5 \times 1.9) = 0.74$

For $y = -1.9$, $w_0 = -1.9 + 3.4 = 1.5$, $s = \tanh(-0.5 \times 1.9) = -0.74$

A single neuron receives two inputs $x_1 = 0.8$ and $x_2 = 1.2$ with weights $w_1 = 1.6$ and $w_2 = 0.6$, respectively. The bias weight is $w_0 = -1.4$. The neuron employs a hyperbolic tangent function of the form $s = \tanh(\alpha y)$, where s is the output signal, y is the activation, and α is a positive parameter. The derivative of s with respect to y is found to be 0.311. Calculate the values of α, y, and s.



Solution

Activation,

$$y = (0.8)(1.6) + (1.2)(0.6) - 1.4$$
$$= \underline{0.6}$$

Derivative,

$$\frac{ds}{dy} = \alpha \operatorname{sech}^2(\alpha y)$$
$$= \alpha[1 - \tanh^2(\alpha y)]$$
$$= 0.311$$

$$\tanh^2(0.6\alpha) = 1 - \frac{0.311}{\alpha}$$

α	1	2	3	4	5	6	$\to\infty$
$\tanh^2(0.6\alpha)$	0.288	0.695	0.896	0.968	0.990	0.997	$\to 1$
$1 - \frac{0.311}{\alpha}$	0.689	0.845	0.896	0.922	0.938	0.948	$\to 1$

From the graph,

$$\alpha = \underline{3}$$

Output signal,

$$s = \tanh(0.6 \times 3)$$
$$= \underline{0.947}$$